

A modified Ginzburg-Landau model for high- T_c superconductivity

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Abstract : A modified Ginzburg-Landau model is proposed incorporating the observed structural transformations. The interplay of the two complex order parameters is expected to explain several of the unusual features of the high- T_c superconductors.

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1. Introduction

The recently discovered oxide superconductors have several characteristics which are not shared by conventional superconductors. The new high- T_c superconductors (HTSC) have small coherence lengths—when extrapolated to zero temperature they seem to be comparable to interatomic separation. The penetration depth of the magnetic field is appreciably larger than the coherence length. These new superconductors are of extreme type II. There are also indications that the superconducting order parameter may have more than one component.

Structural distortions are intrinsic features of these materials—especially La-Ba-Cu-O and Y-Ba-Cu-O compounds. Distortion and lattice softening have been observed in velocity (Bhattacharya *et al* 1987, Horie and Mase 1989), ultra-sound (Bhattacharya *et al* 1988), X-ray (Horn *et al* 1988), neutron scattering (McK Paul *et al* 1987) and Raman scattering (Chrzanowski *et al* 1988) experiments. He *et al* (1989) have shown that Y-Ba-Cu-O system exhibits lattice instability near 250 K and 170 K. Heat flow anomalies and accompanying specific heat anomalies observed close to these temperatures imply that the system undergoes structural phase transitions at these temperatures. The oxide superconductors also possess domain structure at low temperatures (Hervieu *et al* 1987, Pande *et al* 1987). The structural instability can be due to ordering of oxygen vacancies.

Though several mechanisms have been suggested to explain HTSC, there is as yet no consensus on the microscopic mechanism which yields high transition temperature along with the unusual characteristics some of which were mentioned earlier. Consequently, there is a great need for a phenomenological approach on the lines of the celebrated Ginzburg-Landau (GL) theory. Several such GL theories have already been put forth with special modifications relevant to HTSC (Das 1989). The very complicated structural and superconducting behaviours found in these new materials suggests that the two types of transitions may be intimately connected. It was generally believed that there is a close correlation between lattice instability and high temperature superconductivity (Chaturvedi *et al* 1988, Fleming *et al* 1987, Sridhar 1989). Still no GL theory has been formulated which incorporates structural transformations. The present attempt aims at providing such a theory.

Nuclear spin lattice relaxation (Warren *et al* 1987) and specific heat (He *et al* 1989, Laegreid *et al* 1987, Wang Ke-qin *et al* 1989) measurements signal the existence of two kinds of order parameters in high- T_c superconductors. The essential feature of this formalism is the introduction of a complex order parameter for the structural transformation along with the order parameter for the superconducting state involving a phase correlation between the order parameters. This coupling also depends on the oxygen stoichiometry. Although these features cannot be directly explained by the original GL theory, it is an obvious extension of the original idea to include in a single free energy expression, both order parameters and allow them to couple each other.

Recently, some doubts have been raised regarding the relevance of the structural transitions to the onset of high- T_c superconductivity (Torrence *et al* 1989). It should be emphasised that this contribution does not completely eliminate the presence of significant orthorhombic strains in the superconducting state of La-Sr-Cu-O. The model developed in the present contribution is applicable in such a situation also.

2. Formulation of the modified GL theory

In the vicinity of the superconducting transition temperature T_c , the smallness of the superconducting order parameter ψ enables one to expand the free energy F in powers of ψ . Though a similar statement can be made regarding the order parameter φ characterising the structural transformations, one has to introduce a φ^6 term also. The order parameter is taken to be complex as it can be thought of as the expectation value of the phonon annihilation and creation operators (Chaturvedi *et al* 1988). The phonons involved here need not necessarily be linear excitations. The coupling between ψ and φ involves phase interference in an essential manner. One is thus led to the following free energy :

$$\begin{aligned}
F = & \frac{1}{2m} \left| \left(-i\hbar \nabla - \frac{e}{c} A \right) \psi \right|^2 + a_1 |\psi|^2 + \frac{b_1}{2} |\psi|^4 \\
& + \frac{a_2}{2} |\varphi|^2 + \frac{b_2}{4} |\varphi|^4 + \frac{c_2}{6} |\varphi|^6 \\
& + \mu(\psi\varphi^* + \psi^*\varphi) + h^2/8\pi
\end{aligned} \quad (1)$$

This form includes a gauge invariant kinetic energy term for the ψ -field and is useful in studying the response of the superconductor to an external electromagnetic field. In the above the term $h^2/8\pi$ represents the magnetic energy

$$\mathbf{h} = \text{curl } \mathbf{A} \quad (2)$$

Minimising $\mathcal{F} = \int F d^3x$ with respect to the order parameters ψ^*, φ^* and the gauge field in the usual manner one obtains the following equations:

$$\frac{1}{2m} \left(-i\hbar \nabla - \frac{e}{c} A \right)^2 \psi + a_1 \psi + b_1 |\psi|^2 \psi + \mu \varphi = 0 \quad (3)$$

$$a_2 \varphi + b_2 |\varphi|^2 \varphi + c_2 |\varphi|^4 \varphi + 2\mu \psi = 0 \quad (4)$$

$$\frac{1}{4\pi} \text{curl } \mathbf{h} = \frac{1}{c} \mathbf{j} = \frac{e\hbar}{2imc} (\psi \nabla \psi^* - \psi^* \nabla \psi) - \frac{e^2}{mc^2} |\psi|^2 \mathbf{A} \quad (5)$$

A straightforward algebra gives

$$\begin{aligned}
& a_2 \mu^4 [P^2 + (a_1 + b_1 |\psi|^2)] \psi \\
& + b_2 \mu^2 [P^2 + (a_1 + b_1 |\psi|^2)] \psi |\psi|^2 [P^2 + (a_1 + b_1 |\psi|^2)] \psi \\
& + c_2 [P^2 + (a_1 + b_1 |\psi|^2)] \psi |\psi|^4 [P^2 + (a_1 + b_1 |\psi|^2)] \psi \\
& - 2\mu^6 \psi = 0
\end{aligned} \quad (6)$$

where

$$P^2 = \frac{1}{2m} \left(-i\hbar \nabla - \frac{e}{c} A \right)^2$$

For a spatially uniform order parameter ψ in the absence of an electromagnetic field, to the lowest order in $|\psi|^2$ (i.e. to the power zero) one obtains

$$2\mu^2 = a_1 a_2 \quad (7)$$

while to the first order in $|\psi|^2$

$$|\psi|^2 = |\psi_1|^2 = \frac{\mu^2 (2\mu^2 - a_1 a_2)}{(b_1 a_2 \mu^2 + b_2 a_1^2)} \quad (8)$$

In the absence of the structural transformation the order parameter ψ_0 is evaluated to be

$$|\psi|^2 = |\psi_0|^2 = -a_1/b_1 \quad (9)$$

and the thermodynamic critical field h_{c0} is given by

$$h_{c0}^2/8\pi = |\psi_0|^4 \frac{b_1}{2} \quad (10)$$

The effect of structural transformations on ψ_1 can be explicitly written as

$$|\psi_1|^2 = |\psi_0|^2 \alpha, \quad \alpha = \frac{\mu^2[(2\mu^2/a_1) - a_2]}{[a_2\mu^2 + (b_2/b_1)a_1^2]} \quad (11)$$

The thermodynamic critical field in the presence of the structural transformations turns out to be

$$\begin{aligned} (h_{c1}^2/8\pi) = & \left[a_1 |\psi_0|^2 \alpha - \frac{b_1}{2} |\psi_0|^4 \alpha^2 \right] \\ & \frac{1}{2} \left[a_1 + \frac{b_2}{2} |\varphi|^2 + \frac{c_2}{3} |\varphi|^4 \right] |\varphi|^2 \\ & - 2\mu |\varphi| (-\alpha |\psi_0|^2)^{1/2} \cos(\theta_1 - \theta_2) \end{aligned} \quad (12)$$

where θ_1 and θ_2 are the phases of ψ and φ respectively.

Assuming that a_1 and a_2 are regular functions of temperature one can postulate

$$a_1 = a_{10}(T - T_{c0}) \text{ and } a_2 = a_{20}(T - T_d) \quad (13)$$

the superconducting transition temperature T_c in the presence of structural transformations (and in the absence of A) can be written as

$$T_c = \frac{1}{2} \left\{ (T_{c0} + T_d) \pm [(T_d - T_{c0})^2 + 8\mu^2/a_{10}a_{20}]^{1/2} \right\} \quad (14)$$

This expression for the critical temperature for superconducting phase transition contains the effect of coupling the superconductivity and lattice instability. If there is no coupling i.e. when μ vanishes T_c becomes equal to T_{c0} of the ordinary GL theory if the negative sign is taken into account. In all the known oxide superconductors the superconducting phase transition takes place after the structural phase transition and hence the lower sign in eq. (14) may be physically meaningful.

Assuming that the variation of $|\psi|^2$ from $|\psi_1|^2$ as given by eq. (8) is not significant, one can obtain from eq. (6) on setting $A=0$

$$\frac{a_2 \xi^2}{2m(2\mu^2 - a_1 a_2)} \frac{d^2 f}{dx^2} + f - f^3 = 0 \quad (15)$$

where $f = \psi/\psi_1$. This implies that the coherence length ξ is

$$\xi = a_2 \xi^2 / 2m(2\mu^2 - a_1 a_2) \quad (16)$$

Using eq. (5) the penetration depth λ can be calculated to be

$$\lambda = [mc^2 / 16\pi e^2 |\psi_1|^2]^{1/2} \quad (17)$$

3. Discussions

For the case of spatially uniform order parameters in the absence of an electromagnetic field, eqs. (3) and (4) imply

$$2\mu^2 = (a_1 + b_1 |\psi|^2)(a_2 + b_2 |\varphi|^2 + c_2 |\varphi|^4) \quad (18)$$

so that eq. (7) is obtained when $\psi=0, \varphi=0$. Thus μ has to be determined in a self consistent manner.

The calculation could be easily extended to higher orders in $|\psi|^2$. For instance, to the next higher order (upto $|\psi|^4$) one obtains the following expression for the order parameter (denoted by ψ_2).

$$|\psi_2|^2 = -x[1 \pm (1+2|\psi_1|^2/x)^{\frac{1}{2}}] \quad (19)$$

where

$$x = \frac{\mu^2(a_2 b_1 \mu^2 + b_2 a_1^2)}{2a_1^2(c_2 a_1^2 + 3\mu^2 b_1 b_2)} \quad (20)$$

and $|\psi_1|^2$ is given by eq. (8). Choice of the negative sign in eq. (19) seems to be physical. If $|\psi_1|^2 < x$, eq. (19) implies $|\psi_2|^2 \approx |\psi_1|^2$ emphasising the correctness of the approximation made in the previous section.

To summarise, a phenomenological model for high- T_c superconductivity has been proposed. This model is based on the premise that structural transformations, under suitable conditions, may enhance T_c . It should be emphasised that structural transformations in different systems have different effects on superconducting transition and the present model is a general one. Expressions for coherence length and penetration depth are also evaluated and from eqs. (16) and (17) we can see that they also depend on the coupling parameter μ . One may also include a $|\psi|^6$ term (Ginzburg 1987) to take into account the wide fluctuation region. The expressions for critical temperature, coherence length and penetration depth are different from those of the ordinary GL theory. Thus we hope that the present formalism can explain the salient features of high temperature superconductivity.

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